

5. The Fibonacci numbers are defined as follows

$$F_1 = 1, F_2 = 1, F_n = F_{n-1} + F_{n-2} \text{ for } n > 2.$$

Show that if n is an odd integer,

$$F_1 F_n - F_2 F_{n-1} + F_3 F_{n-2} - \dots - F_{n-1} F_2 + F_n F_1 =$$

- A) 0 B) F_n C) F_{n+1} D) ${}_n C_{[n/2]}$.

($[x]$ stands for the greatest integer $\leq x$.)

6. If ${}_n C_r$ stands for the number of ways r objects can be chosen from n different objects,

$${}_n C_0 - {}_{n-1} C_1 + {}_{n-2} C_2 - \dots = \sum_{r=0}^{[n/2]} (-1)^r {}_{n-r} C_r =$$

- A) 0 B) 1 C) $2 \cos \frac{(n+1)\pi}{3}$ D) $\frac{2}{\sqrt{3}} \sin \frac{(n+1)\pi}{3}$

7. If ${}_n C_r$ stands for the number of ways r objects can be chosen from n different objects,

$${}_n C_0 + {}_{n-1} C_1 + {}_{n-2} C_2 + \dots = \sum_{r=0}^{[n/2]} {}_{n-r} C_r =$$

- A) 0 B) F_n C) F_{n+1} D) ${}_n C_{[n/2]}$.

8. If P, Q, R are three points on the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

the maximum value of the area of the triangle PQR is

- A) πab B) $\frac{\pi ab}{2}$ C) $\frac{3\sqrt{3} ab}{4}$ D) unknown

9. Let A, B, C, D be four points on a plane and P be another point. The sum of squares of distances of P from two of the four points equals the sum of distances from the other two, i.e.,

$$(PA)^2 + (PC)^2 = (PB)^2 + (PD)^2$$

- if and only if $ABCD$ is a
- A) parallelogram, B) rectangle, C) square, D) rhombus.
10. It is known that men from the village of St. Mary Mead always lie while the women always tell the truth. In the neighboring village of Chipping Cleghorn, it is just the opposite, men tell the truth and women lie. As a result, there is so much animosity between the residents of the two villages that they rarely if ever intermarry. Louis, an anthropologist meets two married couples in a pub on the edge of the two villages and had the following conversation :

Harry said, "My wife is a liar. And she calls **me** a liar."

Rodney said, "My wife is always truthful."

Jane said, "My husband is not a liar."

Gladys said, "My husband says I am not a liar."

Who are the married couple from Chipping Cleghorn?

- A) Harry & Jane B) Harry & Gladys C) Rodney & Jane
D) Rodney & Gladys