

The Eleventh Annual Catonsville Mathematics Competition

1. If $\{a_n\}$ is the sequence of all positive integers (in increasing order) with all digits even, what is k if $a_k = 2002$?
A) 110 B) 126 C) 200 D) 202
2. If $\{b_n\}$ is the sequence of all positive integers (in increasing order) where the digits from left to right are non-decreasing, what is b_{100} ?
A) 144 B) 178 C) 222 D) 337
3. A polygon is said to be convex if all its interior angles are less than 180° . A convex polygon has area 100 and perimeter 50. If a circle of radius 5 rolls on the outside of the polygon, what is the area enclosed by the curve traced out by the center of the circle?
A) $350 + 25\pi$ B) 150π C) $400 + 25\pi$ D) $450 + 10\pi$
4. The sum of squares of the diagonals of a quadrilateral is 145. What is the ratio of the maximum and minimum possible values of the sum of squares of the sides?
A) 2 B) $2\sqrt{2}$ C) 3 D) 4
5. Mr. Dunkin sold some donuts at a country fair. At the end of the day he noticed a curious thing, that the price (in cents) he charged for each donut was the number with the last two digits of the cube of the total number of donuts he sold (for instance, if he had sold 85 donuts, the price for each donut was 25¢, since $85^3 = 614,125$). Before he went home, he made several phone calls at a pay phone using some of the quarters he got selling donuts. Which of the following amounts could be what was left after making phone calls?
A) \$17.11 B) \$24.37 C) \$29.92 D) \$30.64

6. A spacecraft leaves earth orbit for a distant planet at 12 noon EST today. It travels at 10,000 mph for the first 10,000 miles, at 20,000 mph for the next 20,000 miles, at 30,000 mph for the next 30,000 miles, and so on. It will reach a point 2 million miles from earth at 7:30 AM tomorrow and a point 3 million miles from earth at 12 noon tomorrow. At what time the day after tomorrow will it reach a point 8 million miles from earth?

A) 3:30 AM B) 4:45 AM C) 11:20 AM D) 2:35 PM

7. If N is the product of all positive integers which divide 2002 exactly, what is the sum of the digits of N ?

A) 98 B) 106 C) 126 D) 202

8. What is the 30th digit after the decimal point in $\frac{1}{2002^2}$?

A) 2 B) 4 C) 6 D) 8

9. A and B, two suspects in a fraud case, when interrogated made the following statements.

A said, "I am telling the truth, B is lying."

B said, " Either we are both telling the truth or both lying."

Who is (or are) really lying?

A) only A B) only B C) both A and B D) neither of them

10. The perimeter of a convex (see prob. 3) quadrilateral is 60. Its two longest sides are perpendicular to each other. Which one of the following values could possibly be the area of the quadrilateral.

A) 120 B) 200 C) 240 D) none of them