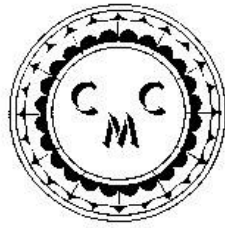


The Twelfth Annual Catonsville Mathematics Competition



1. If ABC is a triangle and D is the point where the internal bisector of the angle A intersects the side BC and E is the point on BC with $BD = CE$, then

$$(AE)^2 - (AD)^2 =$$

- A) $(AB - AC)^2$ B) $|(AB)^2 - (AC)^2|$ C) $2(BC)(DE)$
D) $|(AB) - (AC)|(BC)$
2. If ABC is a triangle with sides of lengths a , b , and c with the usual convention that a is the length of the side opposite angle A and so on and D is the foot of the perpendicular from A on BC and E is the point on BC with $BD = CE$, then

$$(AE)^2 - (AD)^2 =$$

- A) $(b - c)^2$ B) $|b^2 - c^2|$ C) $(b^2 - c^2)^2/a^2$ D) $|b - c|a$
3. A quadrilateral is said to be cyclic if the four vertices lie on a circle. Which of the following statements is not true in general?

- (i) The opposite angles of a cyclic quadrilateral add up to 180° .
(ii) Of all the quadrilaterals with given side-lengths the one with the largest area is cyclic.
(iii) The sum of squares of the lengths of the sides equals the sum of squares of the lengths of the diagonals.
(iv) The perpendicular bisectors of the four sides are concurrent, i.e., they meet at a point.

- A) (i) B) (ii) C) (iii) D) (iv)

4. Let us define a quadrilateral to be a “proper” quadrilateral if the sum of squares of the lengths of the sides equals the sum of squares of the lengths of the diagonals. Which of the following statements is true?

- (i) A quadrilateral is proper if and only if it is a parallelogram.
- (ii) A quadrilateral is proper if and only if it is cyclic.
- (iii) A quadrilateral is proper if the two diagonals intersect at a right angle.
- (iv) A trapezoid is proper if the two sides other than the parallel pair are of equal lengths.

A) (i) B) (ii) C) (iii) D) (iv)

5. Bill and Ted met each other on a hillside going in opposite directions. They were talking about the weather and other things when they noticed a big boulder starting to roll down towards them. Bill immediately started his car and raced downhill away from the boulder while Ted in panic drove his car directly towards the boulder. They both drove at the same constant speed. The boulder crushed Ted’s car and 11 seconds later crushed Bill’s car. Assuming the boulder rolled a distance of $6t^2$ feet in t seconds since it started rolling what was the speed at which Bill or Ted was driving in miles per hour?

A) 33 B) 45 C) 55 D) 66

6. An internet website requires users to logon with a password. The password may be any string of characters from 1 to 20 characters long. There are 94 characters available on a standard keyboard and all of them can be used in a password. Assuming all passwords are equally likely, what is the approximate probability that a password is actually 20 characters long?

A) $1/94$ B) $93/94$ C) $1/20$ D) $19/20$

7. If in the previous question, suppose a password cannot have any character appearing twice and can be a string from 1 to 94 characters long, what is the approximate probability that a password uses all 94 characters assuming all passwords are equally likely?

A) $1/94$ B) $93/94$ C) $0.36788 \cong 1/e$ D) $0.31831 \cong 1/\pi$

8. If N is the product of all positive integers which divide 2003^{2003} exactly, what are the last three digits of N ? (2003 is a prime.)

- A) 027 B) 243 C) 729 D) 999

9. Andy, Billy, Charlie, and Danny made the following statements to a very wise person after consulting with each other:

Andy said, "All of us are telling the truth".

Billy said, "All of us are lying."

Charlie said, "Only two of us are lying."

Danny said, "Only one or two of us are telling the truth."

The wise man was sure about three of them whether they are lying or not. Which one he could not be sure of?

- A) Andy B) Billy C) Charlie D) Danny

10. Two circles of radius 9 cm and 16 cm touch each other at a point A . A line is tangent to them at two other points B and C . The area of the triangle ABC (in cm^2) is

- A) 138.24 B) 168.36 C) 200 D) 240